SUB-MILLIMETER TESTS OF THE GRAVITATIONAL INVERSE SQUARE LAW

E. G. ADELBERGER for the EÖT-WASH GROUP
Center for Experimental Nuclear Physics and Astrophysics
Box 354290, University of Washington
Seattle, WA 98195-4290, USA
E-mail: eric@npl.washington.edu

Sub-mm tests of the gravitational inverse-square law are interesting from several quite different perspectives. This paper discusses work by the Eöt-Wash group performed since the publication of our initial result in February 2001. We find no evidence for short-range Yukawa interactions. Our results provide an upper limit of 200 μm on the size of the largest "extra" dimension, and for the unification scenario with 2 large extra dimensions, set an upper limit of 150 μm on the size of those dimensions.

1 What might be special about gravity at length scales below 1 mm?

Very little is known about gravity at length scales below a few mm [1]. Recently theorists, using several different arguments, have suggested that the unexplored short-range regime of gravitation may hold profound surprises [2,3,4,5], *i.e.* that the gravitational interaction could display fundamentally new behavior in the mm regime.

Many of these arguments are based on the notion, inherent in string or M theory, of more than 3 spatial dimensions. To maintain consistency with a vast body of observations the extra dimensions must be "curled up" in very small regions, usually assumed to be comparable to $R_{\rm P} = \sqrt{G\hbar/c^3} = 1.6 \times 10^{-33}$ cm, or else hidden in some other way [6]. It has recently been noted [2,3] that the enormous discrepancy between natural mass scales of the Standard Model of particle physics ($M_{\rm SM} \approx 1$ TeV) and of gravity (the Planck mass $M_{\rm P} = \sqrt{\hbar c/G} = 1.2\times10^{16}$ TeV) could be eliminated if gravity propagates in *all* the space dimensions while the other fundamental interactions are constrained to the three familiar dimensions. This unification scenario requires that some of the extra dimensions have radii R^* that are large compared to $R_{\rm P}$ with

$$R^* = \frac{\hbar c}{M^* c^2}\left(\frac{M_{\rm P}}{M^*}\right)^{2/n} , \qquad (1)$$

where M^* is the unification scale (usually taken as $M_{\rm SM}$) and n is the number of large extra dimensions. The scenario with $n = 1$ is ruled out by astronom-

ical data. If there are 2 large extra dimensions, R^* must be about 1 mm, and the gravitational inverse-square law (which follows from Gauss's Law in 3 spatial dimensions) will turn into a $1/r^4$-law (Gauss's Law in 5 dimensions) at distances much smaller than R^*.

Completely independent theoretical considerations also suggest that new effects may appear at short distances; string theories predict scalar particles (dilatons and moduli) that generate Yukawa interactions which could be seen in tests of the $1/r^2$ law. If supersymmetry is broken at low energies these scalar particles would produce mm-scale effects [4,7]. Finally, there may be some significance to the observation [5] that the gravitational cosmological constant, $\Lambda \approx 3$ keV/cm^3, deduced from distant Type 1A supernovae [9,10] corresponds to a length scale $\sqrt[4]{\hbar c/\Lambda} \approx 0.1$ mm. These, and other, considerations suggest that the Newtonian gravitational potential should be replaced by a more general expression [8]

$$V(r) = -G\frac{m_1 m_2}{r}(1 + \alpha e^{-r/\lambda}) \ . \tag{2}$$

The simplest scenario with 2 large extra dimensions predicts $\lambda = R^*$ and $\alpha = 3$ or $\alpha = 4$ for compactification on an 2-sphere or 2-torus, respectively [8], while dilaton and moduli exchange could produce forces [4] with α as large as 10^5 for Yukawa ranges $\lambda \sim 0.1$ mm.

2 Experimental Results

In February 2001 we published results of an inverse-square law test [11] obtained with a novel torsion pendulum/rotating attractor instrument. The active component of the pendulum was an aluminum ring with 10 equally-spaced holes bored into it. The pendulum was suspended just above a disk-shaped copper attractor that had 10 similar holes bored into it. As the attractor rotated slowly and uniformly underneath the pendulum, it produced a torque on the pendulum that varied back and forth 10 times for every revolution of the attractor. The attractor actually consisted to two concentric disks each with 10 holes: a thinner upper disk and a thicker lower disk. The holes in the lower disk were rotated by 18 degrees with respect to those in the upper disk so that, if inverse-square-law were correct, the torque on the ring from the lower disk canceled the torque from the upper disk. However, the torque from a short-range interaction could not be canceled simply because the lower disk was too far away to produce a short-range torque on the pendulum. We greatly reduced any electrostatic torques on the pendulum by placing a stationary, tightly stretched 20 μm thick Be/Cu membrane between the pendulum and attractor. Our design had several attractive features:

1. the signal occured at a different frequency than the disturbance (the revolution of the attractor). In this case the signals were at 10ω, 20ω, and 30ω where ω is the attractor rotation frequency.
2. our test bodies were the "missing masses" of the holes in cylindrical rings and disks. This gave us accurately positioned test bodies with planar geometry (optimum because it maximizes the mass that can be placed in close proximity) that could be characterized very precisely.
3. the lower attractor disk that essentially canceled the Newtonian torque greatly reduced our sensitivity to nonlinearities and scale-factor uncertainties in our instrument.

This experiment, which constituted the PhD thesis work of C.D. Hoyle [12], is described in Ref. [11]. The constraint on short-range Yukawa interactions from Ref. [12] is shown in Fig. 1. We encountered a surprising problem in the course of this measurement; for a while looked as if we were observing a substantial violation of the inverse-square law. Despite much effort, we could not account for the apparent violation. So we constructed a second 10-hole torsion pendulum and attractor having holes with different diameters and thicknesses to check the original result and again saw an apparent violation of the $1/r^2$ law. Blayne Heckel finally identified the problem: the commercial computer-controlled micropositioning stage from which the torsion fiber was suspended had a scale factor error–it actually moved only $\approx 98\%$ as far as it indicated. So of course we did not find that $\vec{\nabla} \cdot \vec{g} = 0$; we were using correct distances along $\hat{x}$ and $\hat{y}$ and in incorrect distance along $\hat{z}$! Reference [11] was based on the results from the first (Mark II) instrument. Hoyle has recently reanalyzed the data from the Mark II instrument as well as that from the second (Mark III) instrument. Figure 1 shows the improved constraint from the new analysis of the combined data.

3 Our second-generation instrument

Since the publication of our original results [11], we have made several improvements to our instrument. These were motivated by the recognition that the torque from a very short-range Yukawa interaction with $\lambda \ll s$ (s is the closest attainable pendulum-to-attractor separation) scales as

$$T = \frac{\Delta E}{\Delta \theta} \propto \rho_p \rho_a A \lambda^4 e^{-(s/\lambda)} \ , \tag{3}$$

where E is the interaction energy between the pendulum and attractor, θ is the twist of the pendulum, ρ_p and ρ_a are the densities of the pendulum and attractor, and A is the area of the holes.

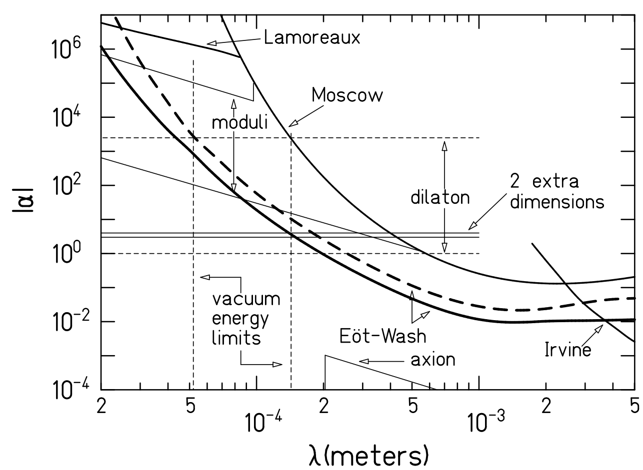

Figure 1: 95% confidence limits on a short-range Yukawa interaction. The heavy dashed line is the Mark II result from Ref. [11], the heavy solid line shows our new analysis of the combined Mark II and Mark III data. The light lines showing some recent predictions are taken from Ref. [1].

1. We increased the torque from a given short-range Yukawa interaction and reduced the Newtonian torque appreciably. This was done by:

    - a new pendulum/attractor design that has two rows of 22 holes (to increase A) with thinner (1 mm thick) pendulum ring and upper attractor disk. The relative sizes of the penduolum and attractor holes was "tweaked" to put most of the power of a Yukawa torque into the fundamental 22ω signal.
    - the pendulum and attractor are both made from molybdenum; this increases the $\rho_p \rho_a$ product by a factor of 4.5.

2. The minimum attainable spacing s is less by at least a factor of two:

    - we reduced the thickness of the conducting membrane to 10 μm.
    - we installed a passive damper that reduced the mean amplitude of the pendulum bounce mode by a factor of 6.

3. We cancelled Newtonian gravity to a higher degree by having thinner active components with more and smaller holes (Newtonian gravity, being

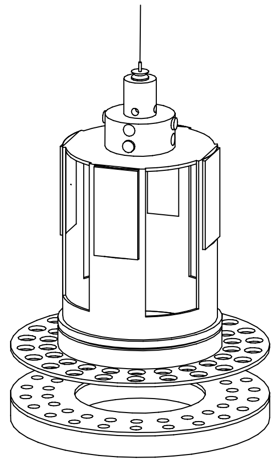

Figure 2: Schematic view of the 22-fold rotationally symmetric Mark V instrument. The active ring of the pendulum is molybdenum and has 44 holes. The two attractor disks are also made from molybdenum. The upper disk has 44 holes while the lower, canceling disk has 22 larger holes situated between the holes in the upper disk. By making the attractor holes have a different diameter than those in the pendulum, we placed more of the Yukawa signal into the fundamental 22ω mode.

long range, tends to average over several holes).

4. We reduced our torque noise by a factor of 6 by improving the autocollimator performance and increasing the pendulum's quality factor Q to ≈ 4000.

The pendulum/attractor of our current Mark V instrument is shown in Fig. 2.

4 Conclusions

We have tested the gravitational inverse-square law at length scales well below 1 mm. We find no evidence for violations. For the proposed scenario with 2 "large" extra dimensions, our negative result corresponds to a 95% confidence upper limit of 150 μm on the size of the 2 dimensions; this implies a unification mass M^* greater than 4.0 TeV. Alternatively, we can use our results to set an upper limit on the size of the *largest* extra dimension, regardless of how many there are (subject only to the assumption that one of the extra dimensions is appreciably larger than the rest). In this case $\alpha = 1$ [13], so that our data imply with 95% confidence that $\lambda < 200$ μm. This upper limit constrains models

that try to explain phenomena such as neutrino oscillations in terms of large extra dimensions.

We expect that, in the next year or so, our torsion-balance scheme for testing the gravitational $1/r^2$ law will provide good results for length scales down to 50 μm. This is less than the diameter of a typical human hair! If we do find evidence for violation of the $1/r^2$ law, we would then develop an instrument to check if the $1/r^2$ violating interaction also violates the weak equivalence principle. This would distinguish between exotic space-time scenarios of extra-dimensions etc. that do not violate the equivalence principle, from exotic particle exchange scenarios that must violate the principle.

However, until evidence for new physics is found, it is clearly better to work on tests of the inverse-square law than on equivalence-principle tests: the $1/r^2$ tests are more general (probing all finite-range effects), and more sensitive (in particle-exchange scenarios the composition-dependence is expected to be a relatively small fractional effect). But testing the gravitational $1/r^2$ law for length scales less than 50 μm will probably require a somewhat different technology. In a planar geometry (optimum because one gets the maximum amount of mass in close proximity) the signal of a short-range Yukawa interaction drops as roughly the 4th power of the Yukawa range while extraneous disturbances stay roughly the same size. This will present an interesting challenge for future experimental work.

Acknowledgments

Jens Gundlach, Blayne Heckel, CD Hoyle, Dan Kapner, Ulrich Schmidt and Erik Swanson all made substantial contributions to the work reported here. We are grateful to Profs. David Kaplan and Keith Dienes for illuminating conversations. This work was supported by the NSF (grant PHY 9970987), and by DOE funding of the Center for Experimental Nuclear Physics and Astrophysics at the University of Washington.

References

1. G.C. Long, H. Chang and J. Price., *Nucl. Phys.* B **529**, 23 (1999).
2. N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* B **429**, 263 (1998).
3. N. Arkani-Hamed, S. Dimopoulos, G. Dvali and N. Kaloper, *Phys. Rev. Lett.* **84**, 586 (2000).
4. S. Dimopoulos and G. Guidice, *Phys. Lett.* B **379**, 105 (1996).
5. R. Sundrum, `hep-th/9708329`.
6. L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (()1999).

7. D.B. Kaplan and M.B. Wise, *J. High Energy Phys.* **8**, 37 (2001).
8. A. Kehagias and K. Sfetsos, *Phys. Lett.* B **472**, 39 (2000). (Our Eq. 2 is valid for extra dimensions of size R^* as long as $r \geq R^*$.)
9. A.G. Riess et al., *Astron. J.* **116**, 1009 (1998).
10. S. Perlmutter et al., *Astrophys. J.* **517**, 565 (1999).
11. C.D. Hoyle et al., *Phys. Rev. Lett.* **86**, 1418 (2001).
12. C.D. Hoyle, PhD thesis, Univ. of Washington (2001).
13. K. Dienes, private communication (2001).